Int. J. Heat Mass Transfer. Vol. 11. pp. 1105 1116. Pergamon Press 1968. Printed in Great Britain

SIMULATION OF A THERMAL REGENERATOR UNDER CONDITIONS OF VARIABLE MASS FLOW

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(*Received* 12 *February* 1968)

Abstract—The naturally periodic characteristics of the regenerative heat exchanger are exhibited by the cyclic variation of the exit gas temperature. If it is required that the regenerator system should supply preheated gas at a constant temperature, then this can be achieved by the inclusion of a by-pass. The proportion of gas passing through the by-pass and the regenerator is then so controlled as to enable the preheated gas to be presented at constant temperature. This paper describes how the differential equations can be solved for such non-linear behaviour. The method developed can also accommodate the temperature dependence of the heat-transfer coefficients and the gas and solid specific heats.

NOMENCLATURE

A, regenerator heating surface area [ft^2];
C, specific heat of heat storing material [Btu/lb degF];
d, wall thickness of chequerwork [ft];
G, operator representing integration process through one cycle of operation;
$\bar{h}$, overall heat-transfer coefficient [$Btu/ft^2 h\,degF$];
h, surface heat-transfer coefficient [$Btu/ft^2 h\,degF$];
L, length of regenerator [ft];
M, mass of heat storing material [lb];
m, number of steps taken in distance direction;
P, length of period;
p, number of steps of integration in time;
S, specific heat of gas [Btu/lb degF];
t, gas temperature [degF];
tci, constant inlet gas temperature during the cooling period [degF];
thi, constant inlet gas temperature during the heating period [degF];
tfx, exit gas temperature at the end of the heating period [degF];
T, mean solid temperature [degF];
$\mathbf{u}$, vector of solid temperatures at beginning of cycle;
W, flow rate of gas [lb/h];
$\mathbf{w}$, vector of solid temperatures at end of cycle;
y, distance from regenerator entrance [ft].

Greek symbols

θ, time [h];
λ, thermal conductivity of heat storing material [Btu/ft h degF];
ϕ, factor which accounts for the inversion of the solid temperature parabolic profile at the regenerator reversals;
α, β, coefficients of difference form of the differential equations;
η, thermal ratio.

Subscripts

r, position in y-direction;
s, position in time, θ;
q, surface of solid;
B, refers to constant blast temperature;
in, inlet to regenerator;
x, exit to regenerator;
H, heating period;
C, cooling period.

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Superscripts

(n), the nth cycle of operation of the regenerator.

INTRODUCTION

The thermal regenerator provides the means whereby heat can be transferred from one gas to another under conditions such as high temperature and a substantial pressure difference between the gases. These conditions make the use of recuperative heat exchangers almost impossible, and certainly very expensive. The regenerator is a device with periodic characteristics and certain difficulties must be overcome when the regenerator is incorporated in a system where continuous heat transfer between the two gases is to be achieved.

The regenerative heat exchanger consists of a honeycomb of heat storing material, often called "chequerwork". In the cycle of regenerator operation heat is transferred from the hot gas passing through the channels of the chequerwork and retained in the heat storing mass. The hot gas is then shut off and cold gas is blown through the channels in the opposite direction and the heat is regenerated from the chequerwork and transferred to the cold gas. At the end of the cycle the cold gas is shut off and the process begins again.

Clearly for there to be a continuous supply of gas heated by the system, a minimum of two regenerators is required so that while one regenerator is supplying heated gas, the other is storing heat from the other gas. Although in this way a constant flow of heated gas is assured, the periodic characteristics of the regenerator are not all eliminated. The temperature of the gas supplied by the regenerator system varies cyclically with time in "saw tooth" fashion, with a period equal to the time during which each regenerator is losing heat to the cold gas, called the "cooling period". This is represented diagrammatically in Fig. 1. The other part of the cycle is called the "heating period".

The Cowper stove is a regenerator used to preheat the blast for an ironmaking furnace. It is easier to operate the blast furnace when the blast temperature is constant. It is common for the "saw-tooth" blast temperature to be smoothed out to a constant temperature by including a by-pass main in the stove system, as illustrated in Fig. 2.

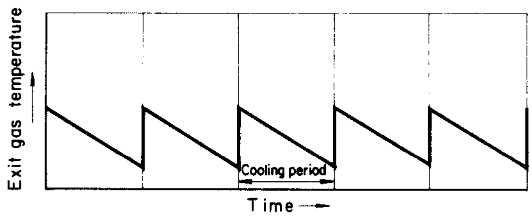


FIG. 1. The variation of blast temperature from Cowper stove without by-pass main.

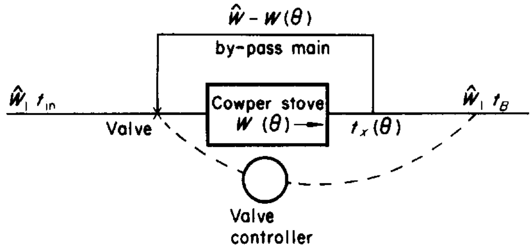


FIG. 2. Blast temperature control system.

The furnace is supplied with hot blast at a constant mass flow rate of $\hat{W}$ at a constant blast temperature of t_B. This means that a regenerator system operated in this way behaves as though it were a recuperative heat exchanger without any periodic characteristics.

The mass flow of blast through the regenerator varies with time θ, (see Fig. 3). When $W(\theta)$ is

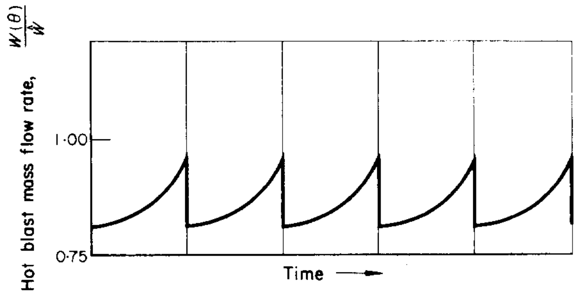


FIG. 3. Time variation of air volume passing through stoves.

passing through the regenerator itself, $\hat{W} - W(\theta)$ goes through the by-pass. The proportion of air passing through the regenerator is controlled so that the temperature of the blast remains constant.

Although the system supplies preheated gas at a constant mass flow rate, the regenerators themselves behave under conditions of variable mass flow. In a previous paper [1], the author described a method of simulating regenerators for when the mass flow rates of the heating and cooling gases do not vary throughout each heating and cooling period. This paper describes how variable mass flow of the cold and/or hot gases, under conditions such as those described above, can be incorporated into a theoretical model of a regenerator. The method described is a general one and can be used to solve most if not all of the non-linear problems of regenerator calculation by a digital computer. In particular, the procedure extends to include the temperature dependence of the thermal properties of the gas and of the material of the chequerwork. The method has been programmed for the Manchester University ATLAS Computer.

THE DIFFERENTIAL EQUATIONS

The differential equations which are used to describe the thermal behaviour of the regenerator are

$$\bar{h}A(T - t) = WSL\frac{\partial t}{\partial y} \quad (1)$$

$$\bar{h}A(t - T) = MC\frac{\partial T}{\partial \theta}. \quad (2)$$

They are based on the following assumptions:

(i) The effect of the reversals can be neglected, that is the rapid gas temperature transients which are associated with the residual gas in the regenerator as it is driven out by the gas flow in the opposite direction immediately after the reversal can be ignored.

(ii) The heat transfer between gas and solid can be represented in terms of an overall heat-transfer coefficient, $\bar{h}$, relating gas temperature t to mean solid temperature, T. This overall coefficient is connected to the surface heat-transfer coefficient, h, by the relation

$$h(T_q - t) = \bar{h}(T - t)$$

where T_q is the surface solid temperature. Hausen [2], showed that h and $\bar{h}$ are related by the equation

$$\frac{1}{\bar{h}} = \frac{1}{h} + \frac{d\phi}{6\lambda}$$

where d is the thickness of the walls of the chequerwork, λ is the thermal conductivity of the chequerwork material, and ϕ is a factor which accounts for the inversion of the solid temperature parabolic profile at the regenerator reversals.

(iii) The heat capacity of the gas in the channels of the matrix at any instant is small relative to the heat capacity of the chequerwork and can be neglected.

(iv) Longitudinal thermal conductivity is neglected.

(v) The entrance gas temperature in both periods remains constant.

This last assumption defines one of the boundary conditions. The other boundary condition is that the solid temperatures at the end of the heating–cooling period are the same as at the beginning of the succeeding cooling–heating period.

The direction y is always mentioned measured in the direction of gas flow with the origin at the current regenerator entrance. Hence this latter boundary condition is expressed as

$$T'(y, 0) = T(L - y, P)$$

where the prime refers to the succeeding period. The value of $T(0, y)$ for $0 \leqslant y \leqslant L$ at the beginning of the first cycle is defined arbitrarily.

FINITE DIFFERENCE SOLUTION

The partial differential equations are expressed in finite difference form and integrated by a step by step process using the trapezoidal method. The solid and gas temperatures are calculated at equally spaced heights in the regenerator, Δy apart, and at equally spaced intervals of time $\Delta\theta$, as indicated in Fig. 4. (The subscripts r and s in this diagram refer to distance and time respectively.)

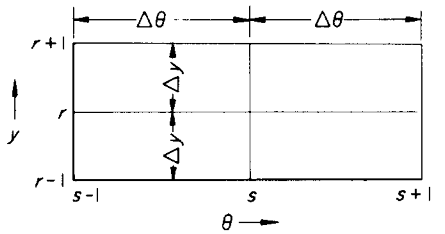


FIG. 4. Section of space–time grid for difference representation of differential equations.

The difference forms of the differential equations (1) and (2) are

$$t_{r+1,s} = t_{r,s} + \frac{\Delta y}{2}\left\{\left(\frac{\partial t}{\partial y}\right)_{r+1,s} + \left(\frac{\partial t}{\partial y}\right)_{r,s}\right\} \quad (3)$$

$$T_{r,s+1} = T_{r,s} + \frac{\Delta\theta}{2}\left\{\left(\frac{\partial T}{\partial\theta}\right)_{r,s+1} + \left(\frac{\partial T}{\partial\theta}\right)_{r,s}\right\} \quad (4)$$

where

$$\left(\frac{\partial t}{\partial y}\right)_{r,s} = \left\{\frac{\bar{h}A}{WSL}(T - t)\right\}_{r,s}$$

and

$$\left(\frac{\partial t}{\partial\theta}\right)_{r,s} = \left\{\frac{\bar{h}A}{MC}(t - T)\right\}_{r,s}$$

The flow rate W varies with time in the operation of a regenerator with a by-pass main. The convective heat-transfer coefficient is a function of this flow rate and therefore also varies with time. If the temperature dependence of the heat-transfer coefficients and the gas and solid specific heats is considered, $\bar{h}$, S and C vary with position in the regenerator and with time. It is possible to define $\alpha_{r,s}$ and $\beta_{r,s}$ as

$$\alpha_{r,s} = \frac{\bar{h}_{r,s}A}{W_s S_{r,s} L}\frac{\Delta y}{2}$$

and

$$\beta_{r,s} = \frac{\bar{h}_{r,s}A}{MC_{r,s}}\frac{\Delta\theta}{2}.$$

The difference equations can be re-written in the form

$$t_{r+1,s} = A1_{r,s}t_{r,s} + A2_{r+1,s}T_{r+1,s} + A3_{r,s}T_{r,s} \quad (5)$$

$$T_{r,s+1} = B1_{r,s}T_{r,s} + B2_{r,s+1}t_{r,s+1} + B3_{r,s}t_{r,s} \quad (6)$$

where

$$A1_{r,s} = \frac{1 - \alpha_{r,s}}{1 + \alpha_{r+1,s}}, \quad A2_{r+1,s} = \frac{\alpha_{r+1,s}}{1 + \alpha_{r+1,s}},$$

$$A3_{r,s} = \frac{\alpha_{r,s}}{1 + \alpha_{r+1,s}}, \quad B1_{r,s} = \frac{1 - \beta_{r,s}}{1 + \beta_{r,s+1}},$$

$$B2_{r,s+1} = \frac{\beta_{r,s+1}}{1 + \beta_{r,s+1}}, \quad B3_{r,s} = \frac{\beta_{r,s}}{1 + \beta_{r,s+1}}.$$

Equation (6) involves a prior knowledge of $t_{r,s+1}$. However, $t_{r,s+1}$ can be calculated by equation (5) and

$$t_{r,s+1} = A1_{r-1,s+1}t_{r-1,s+1} + A2_{r,s+1}T_{r,s+1} + A3_{r-1,s+1}T_{r-1,s+1}.$$

This is substituted into equation (6) and yields

$$T_{r,s+1} = K1_{r,s}T_{r,s} + K2_{r-1,s+1}T_{r-1,s+1} + K3_{r,s}t_{r,s} + K4_{r-1,s+1}t_{r-1,s+1} \quad (7)$$

where

$$K1_{r,s} = B1_{r,s}/X$$

$$K2_{r-1,s+1} = A3_{r-1,s+1}B2_{r,s+1}/X$$

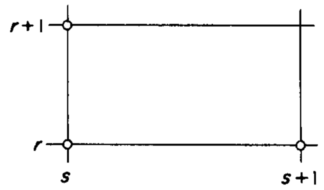
$$K3_{r,s} = B3_{r,s}/X$$

$$K4_{r-1,s+1} = A1_{r-1,s+1}B2_{r,s+1}/X$$

and where

$$X = 1 - A2_{r,s+1}B2_{r,s+1}.$$

At any stage of the integration process, the solid and gas temperatures are known at the points indicated in Fig. 5, at (r, s), $(r, s + 1)$ and



At any juncture of the solution of the differential equations, the gas and solid temperatures are known at points shown.

FIG. 5.

$(r + 1, s)$ and it is required to calculate the temperatures at $(r + 1, s + 1)$.

The solid temperature $T_{r+1,s+1}$ is calculated using equation (7) applied at this position.

$$T_{r+1,s+1} = K1_{r+1,s}T_{r+1,s} + K2_{r,s+1}T_{r,s+1} + K3_{r+1,s}t_{r+1,s} + K4_{r,s+1}t_{r,s+1}. \quad (8)$$

The gas temperature $t_{r+1,s+1}$ can then be evaluated in a similar way using equation (5)

$$t_{r+1,s+1} = A1_{r,s+1}t_{r,s+1} + A2_{r+1,s+1}T_{r+1,s+1} + A3_{r,s+1}T_{r,s+1}. \quad (9)$$

The process is continued over the whole period for $s = 0, 1, 2, \ldots p - 1$ where $p\Delta\theta = P$ and over the whole length of the regenerator for $r = 0, 1, 2, \ldots, m - 1$ where $m\,\Delta y = L$. At this stage the counterflow reversal boundary condition is applied and integration over the succeeding period continues. The values of the constants $A1$, $A2$, $A3$, $K1$, $K2$, $K3$ and $K4$ for all the values of r and s will of course in all probability be different for the two periods, if only because, for example, the specific heats of the heating and cooling gases are different.

THE INTEGRATION PROCESS

The description thus far of this process of integrating the differential equations has glossed over the fact that it has been presumed that at any position r in the regenerator and at any point s in time, W is known and that it is possible to calculate the heat-transfer coefficient $\bar{h}$ and the specific heats S and C. This, of course, is not true; for example, the specific heat of the gas $S_{r,s}$ is a function of the gas temperature $t_{r,s}$, and the specific heat of the chequerwork material $C_{r,s}$ is a function of the solid temperature $T_{r,s}$ and these temperatures are unknown. Indeed, the purpose of the exercise is to compute $t_{r,s}$ and $T_{r,s}$.

A number of methods exist for tackling this problem. One appears in the appendix of the earlier paper [1] to which reference has already been made. This consists, in essence, of solving sets of simultaneous non-linear algebraic equations, which represent the difference form of the differential equations . This method is laborious to program and suffers from the particular disadvantage that a computer program written to deal only with temperature dependent gas specific heat, for example, cannot readily be altered to cope with time varying flow rate. The method described here does not suffer from this disadvantage. Once the integration process, using equations (8) and (9) has been programmed, different subroutines can be built into the same computer program which calculate the coefficients $A1, A2, \ldots, K1, K2, \ldots$, (for all r, s) for the different non-linear problems which are required to be solved.

The integration process can be regarded conveniently as some operation G upon the vector $\mathbf{u}$ of solid temperatures at the beginning of the cycle,
where

$$\mathbf{u} = \{T_{0,0}, T_{1,0}, T_{2,0}, \ldots, T_{m,0}\}.$$

The result of this operation is the calculation of the vector $\mathbf{w}$ of solid temperatures at the end of the cycle,
where

$$\mathbf{w} = \{T_{0,p}, T_{1,p}, T_{2,p}, \ldots, T_{m,p}\}.$$

This integration process can be represented by the equation

$$\mathbf{w}^{(n)} = G^{(n)}\mathbf{u}^{(n)}$$

where the superscript (n) denotes the nth cycle. The operation $G^{(n)}$ requires the values of $A1_{r,s}$, $A2_{r,s}, \ldots, K1_{r,s}, K2_{r,s} \ldots$, (for all r, s) for the nth cycle.

The regenerator cycles naturally to dynamic equilibrium. In a similar way the mathematical model of the regenerator cycles to equilibrium, that is the system converges to the solution

$$\mathbf{u} = G\mathbf{u}.$$

It is possible to make some initial estimate of the solid and gas temperatures at equilibrium by use of a simplified integration process [1, 3, 4], employing estimated mean values of $\bar{h}$, S, C and W. From these values of the gas and solid temperatures it is possible to obtain approximate values of $A1_{r,s}$, $A2_{r,s}, \ldots, K1_{r,s}, K2_{r,s} \ldots$ for all r, s, that is an approximate value of $G^{(1)}$, and employing this, $\mathbf{w}^{(1)}$ is computed

$$\mathbf{w}^{(1)} = G^{(1)}\mathbf{u}^{(1)}.$$

Employing the freshly calculated values of $T_{r,s}$ and $t_{r,s}$ a new estimate of G is obtained. This is regarded as $G^{(2)}$ and $\mathbf{w}^{(1)}$ is considered to be the initial solid temperature $\mathbf{u}^{(2)}$ at the beginning of the second cycle. The whole integration process becomes the series of operations listed below:

$$\mathbf{w}^{(1)} = G^{(1)}\mathbf{u}^{(1)}$$

$$\mathbf{w}^{(2)} = G^{(2)}\mathbf{w}^{(1)}$$

$$\mathbf{w}^{(3)} = G^{(3)}\mathbf{w}^{(2)}$$

$$\ldots$$

$$\ldots$$

$$\mathbf{w}^{(n)} = G^{(n)}\mathbf{w}^{(n-1)}$$

This iterative procedure is allowed to continue until $|\mathbf{w}^{(n)} - \mathbf{w}^{(n-1)}| = |\mathbf{u}^{(n+1)} = \mathbf{u}^{(n)}|$ becomes very small. At this stage, it is regarded that $\mathbf{w}^{(n)}$ $(= \mathbf{u}^{(n+1)})$ has converged to $\mathbf{u}$ and $G^{(n)}$ has converged to G, and that the cyclic equilibrium condition has been reached where

$$\mathbf{u} = G\mathbf{u}.$$

In mathematical terms, this depends on the order of convergence of the natural process of reaching equilibrium being greater than the order of convergence of $G^{(t)}\mathbf{u}^{(t)}$ to $G\mathbf{u}$. This is extremely difficult to prove. Nevertheless, this process has been programmed for the Manchester University Atlas Computer and it has been found that it converges quite rapidly.

This method is clearly quite powerful and is applicable to a wide range of forced oscillation problems where the natural process of reaching a dynamic equilibrium can be used to "carry along" or to accelerate the convergence of an operator $G^{(n)}$ to G.

THE VARIABLE MASS FLOW PROBLEM

The by-pass main method of operating a regenerator provides a means of supplying a preheated gas at a constant mass flow rate of W and at a constant temperature t_B. When $W(\theta)$ is passing through the regenerator, $\hat{W} - W(\theta)$ is passed through the by-pass main. The preheated gas is mixed with the cold air by-passed round the regenerator and the proportion of gas passing through the regenerator at any instant is so controlled that the temperature of the mixed gases, t_B does not change.

This imposes a continuous rate of heat regeneration from the regenerator system equal to $\hat{W}S(t_B - t_{in})$ where t_{in} is the inlet temperature of the cold gas. If the system is perfectly controlled, at any instant the heat balance equation given below will be satisfied.

$$W(\theta)(t_x(\theta) - t_{in}) = \hat{W}(t_B - t_{in})$$

if the temperature dependence of the gas specific heat is ignored. $t_x(\theta)$ is the exit temperature of the unmixed blast at time θ.

In the heating period it is assumed that the mass flow rate of gas does not vary and that the inlet temperature of the gas is constant.

In the computer program developed, an initial estimate of chequerwork temperature profile at the beginning of the heating period is calculated for cyclic equilibrium assuming that the flow rate through the regenerator is constant and equal to $\hat{W}$ in the cooling period.

The variable mass flow $W(P)$ of the gas at the end of the cooling period is then fixed so that $W(P)/\hat{W}$ is constant and in the calculations performed set equal to and fixed to be 0·95. The mass flow rate is initially assumed to vary linearly from, say $W(\theta)/\hat{W} = 0{\cdot}75$ to $0{\cdot}95$ in the cooling period. The convective heat-transfer coefficients are then calculated at each point s in time, at intervals of $\Delta\theta$. It is assumed that the gas specific heat, the chequerwork specific heat and thermal conductivity are constant throughout the period and that the heat-transfer coefficients are functions of the mass flow rate of the gas only.

The coefficients $A1$, $A2$, $A3$, $K1$, $K2$, $K3$ and $K4$ of the difference equations (8) and (9) for the cooling period can now be calculated. The corresponding coefficients for the heating period are calculated once for the whole computation since it is assumed that the mass flow rate of the gas in the heating period is constant and fixed.

The integration of the differential equations for the first cycle is now computed and the exit temperatures $t_x(\theta)$ in the cooling period are calculated. The mixed constant gas temperature t_B at the end of the cooling period is next calculated using the heat balance

$$W(P)\,(t_x(p) - t_{in}) = W(t_B - t_{in})$$

which becomes

$$t_B = t_{in} + 0{\cdot}95\,(t_x(p) - t_{in})$$

since $W(P)/\hat{W}$ is fixed to be 0·95.

The flow rates $W(0)$, $W(1)$, ..., $W(s)$, ..., $W(p-1)$ together with the corresponding heat-transfer coefficients $\bar{h}(0)$, $\bar{h}(1)$, ..., $\bar{h}(s)$, ..., $\bar{h}(p-1)$ are recalculated so that

$$W(\theta) = \hat{W}(t_B - t_{in})/(t_c(\theta) - t_{in})$$

for values of θ which correspond to the values $s = 0, 1, 2, \ldots, p-1$.

The new values of $A1$, $A2$, $A3$, $K1$, $K2$, $K3$ and $K4$ are recalculated and the next cycle is computed.

The process is continued until the difference between successive values of the mixed gas temperature t_B becomes small, or until the operator $G^{(n)}$ converges to G and the vector $\mathbf{u}^{(n)}$ converges to $\mathbf{u}$.

By this means, the constant mixed gas temperature at cyclic equilibrium is calculated together with the time variation of the gas mass flow rate through the regenerator in the cooling period. The exit gas temperatures in the heating period and the chequerwork temperature are also computed.

THE BOUNDARY VALUE PROBLEM

The method described above to solve the differential equations relates to the "open ended problem". Given a regenerator specification and operating conditions, what blast temperature can be achieved, how does the flow rate vary in the cooling period, how do the gas exit temperatures vary with time in the heating and cooling periods?

However, in computing regenerator thermal behaviour, very often the question is asked, "What is the maximum blast temperature which can be achieved such that the exit waste gas temperature at the regenerator chimney stack never exceeds a specified temperature?". This is a "closed ended" problem or boundary value problem. The regenerator physical details and/or the operating conditions are not completely specified. Instead, restrictions are imposed on the possible solution to the problem; in the case mentioned, restrictions are imposed on the highest temperature which the exit waste gas temperature in the heating period can be permitted to attain.

However, great care must be exercised in the selection of the boundary value problem to be solved. If the problem is posed, "What is the minimum cycle time such that the highest blast temperature will be attained?", then the meaningless zero cycle time is the solution. On the other hand, for otherwise completely specified physical details and operating conditions, if the problem is posed "What area and mass of chequerwork (the area/mass ratio being fixed) is required for a specified blast temperature?", then sometimes no solution will exist. The graphs in Fig. 6 illustrate this difficulty.

A modification to the computer program has been developed to solve the first mentioned boundary value problem. Here for specified operating conditions and regenerator physical

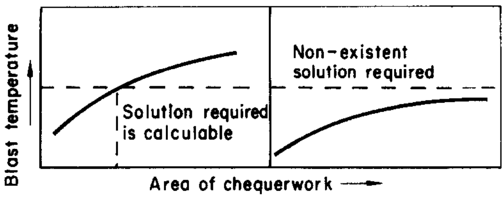


FIG. 6. Examples of the existence and non-existence of solutions to boundary value problems.

details, the waste gas rate is calculated such that the maximum exit gas temperature in the heating period is equal to some pre-determined value. Simultaneously, of course, the blast temperature achieved, the variation of the air flow through the stove in the cooling period etc. are calculated for these conditions. From thermodynamic considerations, it will be seen that such a solution is possible only if

$$tci < tfx < thi$$

where tci, constant inlet gas temperature (cooling);
thi, constant inlet gas temperature (heating);
tfx, final exit gas temperature (heating).

The exit waste gas temperature can never exceed the hot inlet gas temperature or be lower than the cold inlet gas temperature.

The method adapted here to solve this problem is called "Regula Falsi", or the method of false position.

The obtaining of the solution to the differential equations

$$\mathbf{u} = G\mathbf{u}$$

described earlier in this paper, can be regarded as a non-linear function of waste gas flow rate Wg, say $f(Wg)$ where $f(Wg) = tfx$. If the final exit gas temperature in the heating period is specified to be TFX, the problem becomes one of solving the non-linear equation

$$f(Wg) - TFX = 0.$$

Suppose $Wg^{(n-1)}$ and $Wg^{(n)}$ are the $(n-1)$th and nth estimates of Wg and $Z^{(n-1)}$ and $Z^{(n)}$ are the corresponding values of $f(Wg) - TFX$. The $(n+1)$th estimate of Wg is obtained by calculating the intersection (through the Wg axis) of the chord which passes through the nth and $(n-1)$th points. It can be shown that by similar triangles, (see Fig. 7)

$$\frac{Wg^{(n+1)} - W^{(n-1)}}{Wg^{(n+1)} - W^{(n)}} = \frac{Z^{(n-1)}}{Z^{(n)}}.$$

This leads to

$$Wg^{(n+1)} = \frac{Wg^{(n-1)}Z^{(n)} - Wg^{(n)}Z^{(n-1)}}{Z^{(n)} - Z^{(n-1)}}.$$

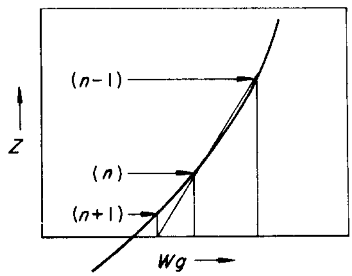


FIG. 7. Regula Falsi. Z_{n+1} is calculated from Z_n and Z_{n-1}.

This iterative process is continued until $|Z^{(n)}|$ becomes very small. This process requires repeated solutions of the differential equations, represented by

$$\mathbf{u} = G\mathbf{u}.$$

For the initial calculation of $Z^{(1)}$ from $Wg^{(1)}$, the initial solid temperatures are estimated using a simplified method of regenerator calculation, mentioned above. The initial estimate of variation of flow rate in the cooling period is a simple linear variation with time. The variation of flow rate enables the first estimate of the operator G to be calculated.

However, for every subsequent calculation of $Z^{(n)}$ from $Wg^{(n)}$, the initial solid temperatures and the variation of the flow rate in the cooling period are taken to be those computed for the $(n-1)$th iteration. As $|Z|$ approaches zero, the changes between subsequent values of Wg are small. It follows that the approximations to the vector of solid temperature **u** and the operator G improve in the successive Z, Wg iterations. This accelerates the computation.

DIMENSIONLESS PARAMETERS

In the linear problem, that is when the heat-transfer coefficients and the gas and the solid thermal properties are regarded as invariant with respect to location in the regenerator and to time, and the flow rate is constant with time, the differential equations (1) and (2) can be simplified by the replacements:

$$\xi = \bar{h}Ay/WSL \quad \text{and} \quad \eta = \bar{h}A\theta/MC.$$

The equations then become

$$\frac{\partial t}{\partial \xi} = T - t \tag{10}$$

$$\frac{\partial T}{\partial \eta} = t - T. \tag{11}$$

When $y = L$ and $\theta = P$ for each period of the operating cycle, the dimensionless parameters "reduced length", Λ, and "reduced period", Π, are defined as:

$$\Lambda = \bar{h}A/WS \quad \text{and} \quad \Pi = hAP/MC.$$

However, in a non-linear problem, this is not possible since, as in the case considered here, the flow rate is time dependent and therefore the values of the descriptive dimensionless parameters vary with time. In the linear problem, a problem is uniquely defined by the dimensionless parameters. In the non-linear problem, the dimensionless parameters are not especially helpful since it is the time variation of the dimensionless parameters which uniquely defines the problem to be solved. Further in the case considered here, it is implicitly required to compute for the regenerator configuration under consideration, how the dimensionless parameters vary with time.

For the variable flow problem, it will be seen that if $\bar{h}$ is regarded as being approximately proportional to $W^{0.8}$, then the time variation of the dimensionless parameters can be roughly represented in the form

$$\Lambda(\theta) = k_1 W(\theta)^{-0.2}$$

$$\Pi(\theta) = k_2 W(\theta)^{0.8}$$

where k_1 and k_2 are constants.

PRACTICAL CALCULATIONS

To illustrate the usefulness of this method of regenerator calculation, the effect of cycle time upon performance has been investigated for a regenerator of Cowper stove dimensions. It is in blast furnace practice that the bypass main is used to control the blast temperature delivered by the regenerator system.

A three regenerator system is considered, one regenerator delivering preheated air at a constant (blast) temperature, while the other two are heated up by a gas of entrance temperature 2200°F. The ratio of the length of the heating period to that of the cooling period is thus 2:1.

Table 1

Regenerator details		
Heating surface area, A	245000 ft^2	
Mass of chequerwork, M	2464000 lb	
Wall thickness of chequerwork, d	0·166 ft	
Chequer work material		
Thermal conductivity, λ	0·7 Btu/ft h degF	
Specific heat, C	0·25 Btu/lb degF	
Operating conditions		
	Heating period	Cooling period
Flow rate of gas, W	154375·2	290556 lb/h
Gas specific heat, S	0·28	0·251 Btu/lb degF
Gas entrance temperature	2200	200 degF

Table 2

	case 1	case 2	case 3
Constant flow			
Thermal ratio, η			
Heating period	0·778	0·780	0·781
Cooling period	0·922	0·925	0·926
Variable flow			
Heating period	0·726	0·735	0·739
Cooling period (for unmixed gas)	0·944	0·943	0·942
Cooling period (for mixed gas)	0·861	0·871	0·876

Total cycle times of 4·5, 3 and 2·25 h are considered and these are called case 1, case 2 and case 3 respectively.

The behaviour of the regenerator was computed for the constant flow condition and then for when the bypass main was employed to control the blast temperature.

It will be seen that the thermal ratio for the heating period is lower for variable flow operation than for constant flow. In other words, variable flow operation with a bypass main degrades the effectiveness of the regenerator to transfer heat from the hot to the cold gas. Comparing the cooling period thermal ratios reveals that, although that ratio for unmixed variable flow gas is highest, this is degraded by the bleeding of cold air from the bypass into the hot blast from the regenerator, so that the thermal ratio for the mixed blast becomes lower than that for the constant flow problem.

Regenerator performance improves as cycle time is reduced and this is confirmed by these figures.

The variation of flow rate of gas through the regenerator for this mode of operation is interesting. For the three cases considered, the variation is presented in graphical form in Fig. 8.

The solid temperature distribution in the regenerator at the end of the two periods of operation are presented in Fig. 9.

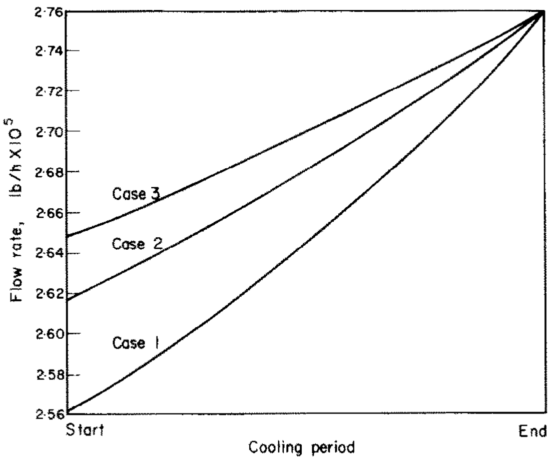


FIG. 8. Variation of flow rate of gas through regenerator in cooling period.

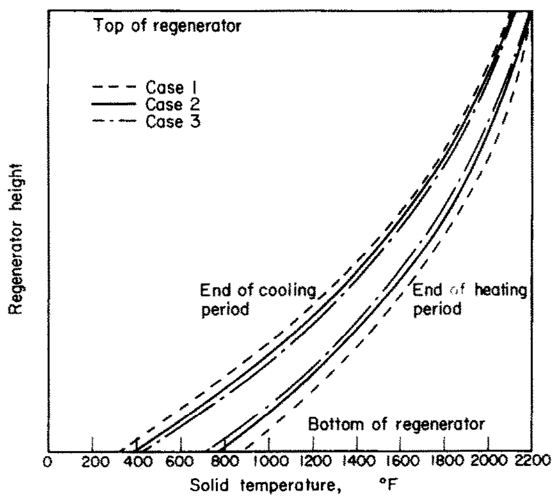


FIG. 9. Solid temperature distribution in regenerator at the end of both periods of operation.

BOUNDARY VALUE PROBLEM CALCULATIONS

The method has been used to calculate the

Table 3

Final waste gas temperature at exit (°F)	Blast temperature (°F)	Waste gas rate (s.c.f.m.)
400	1464	26265
500	1567	29489
600	1646	32447
700	1709	35318
800	1762	38218

Table 4. Regenerator details and operating conditions

Heating surface area, A	206000	ft^2
Chequer thickness, d	0·104	ft
Chequer mass, M	1858760	lb
Chequer material		
Thermal conductivity, λ	0·7	Btu/ft degF
Specific heat, C	0·3	Btu/lb degF
Blast volume, W	90000	s.c.f.m.
Blast specific heat, S''	0·251	Btu/lb degF
Waste gas specific heat, S'	0·280	Btu/lb degF
Gas entrance temperature		
Heating period	2160	degF
Cooling period	200	degF

Cycle time, 2 h on gas, 1 h on blast.

maximum mixed blast temperature attainable for the restriction that the waste gas temperature should not exceed 400, 500, 600, 700 and 800°F. Set out in Table 3 are these blast temperatures, together with corresponding calculated waste gas rate in the heating period. The physical details of the regenerators and the operating conditions are also tabulated.

CONCLUSIONS

An earlier paper [1] by the author explained how the differential equations describing the thermal behaviour of regenerators can be solved using finite difference methods. Consideration was given to the "linear" case where the thermal properties of the gases and chequerwork solid were temperature-independent and where the gas flow rates did not vary with time. Two important developments are described in this paper. The non-linear problem has been solved, which deals in this case with the flow variation imposed in the cooling period when some means of control of the blast temperature is included in a regenerator system. The method of Regula Falsi has been employed to tackle problems of the boundary value type.

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Résumé—Les caractéristiques périodiques naturelles de l'échangeur de chaleur par récupération sont montrées par la variation cyclique de la température de sortie du gaz. Si l'on demande que le système du récupérateur fournisse du gaz préchauffé à température constante, alors ceci peut être réalisé en incorporant un court-circuit. La proportion de gaz passant à travers le cours-circuit et le récupérateur est alors contrôlée de façon à permettre au gaz préchauffé de se présenter à température constante. Cet article décrit comment les équations différentielles peuvent être résolues pour un comportement non-linéaire de ce type. La méthode exposée peut également tenir compte de la dépendance de la température des coefficients de transport de chaleur et des chaleurs spécifiques du gaz et du solide.

Zusammenfassung—Die natürlichen periodischen Charakteristika eines regenerativen Wärmeübertragers zeigen sich in der zyklischen Änderung der Austrittsgastemperatur. Soll das Temperatursystem vorgeheiztes Gas mit konstanter Temperatur liefern, so kann das über einen By-Pass geschehen. Der Anteil des durch den By-Pass strömenden Gases und der Regenerator werden dann so kontrolliert, dass das Gas mit konstanter Temperatur anfällt. Diese Arbeit beschreibt, wie die Differentialgleichungen für ein derartiges nicht-laminares Verhalten gelöst werden können. Die entwickelte Methode kann auch die Temperaturabhängigkeit der Wärmeübergangskoeffizienten und der spezifischen Wärmen von Gas und Festkörpern berücksichtigen.

Аннотация—Натуральные периодические характеристики теплового регенератора показывают циклическое изменение температуры газа на выходе. Если необходимо, чтобы система регенератора подавала подогретый газ при постоянной температуре, этого можно достигнуть включением обводного канала. Соотношение газа, проходящего через обводной канал и регенератор, контролируется таким образом, чтобы подогертый газ имел постоянную температуру. В статье описано, как могут быть решены дифференциальные уравнения, орисывающие объект со столь нелинейной характеристикой. Развитый метод может также учитывать темлературную зависимость коэффициентов теплообмена и удельную теплоту газов и твердых тел.